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A Model Predictive Control Algorithm for Cost Optimization of a Building in Hybrid Heating System

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ABSTRACT

With the increasing availability and affordability of building-integrated Heat Pumps (HPs), the number of heat pumps installed in residential buildings has risen significantly in recent years. When coupled with conventional District Heating (DH) systems in a hybrid setting, HPs provide higher energy reliability and cost-effective solutions for domestic heating. The operation of such systems, however, requires a sophisticated control system that simultaneously considers the dynamics of energy pricing and building energy needs. In this paper, we propose a nonlinear economic model predictive control to determine the optimal share for a hybrid DH-HP heating system. A resistor-capacitor thermal building model is utilized to capture the system dynamics. The results indicate that the proposed controller in the hybrid DH-HP system has a cost saving between 29% and 57% compared to the baseline scenario.

Keywords: Model predictive control, Cost optimization, Hybrid energy system, Heat pump, District heating, Nonlinear

NOMENCLATURE

Subscripts	
t	Index of time
Superscripts	
min/max	Minimum/Maximum boundaries
Symbols	
T_{in}	Building indoor temperature
T_{out}	Building outdoor temperature
T_e	Building envelope temperature
ϕ_{DH}	Heating power of district heating
ϕ_{HP}	Heating power of heat pump
ϕ_{sol}	Solar irradiance per area
P_{HP}	Electric power of heat pump
η_{DH}	District heating efficiency
COP_{HP}	Heat pump coefficient of performance
ε_k	Slack variable
w_k	Weighting factor of the slack variable
C_{in}	Inner mass thermal capacitance
C_e	Envelope thermal capacitance
A_w	Total area of windows
A_e	Total area of exterior walls
G_w	Solar heat gain of windows
G_e	Solar heat gain of walls
R_{in}	Inner mass thermal resistance
R_e	Envelope thermal resistance
R_w	Windows thermal resistance

1. INTRODUCTION

District Heating (DH) systems are the primary means of supplying heat to urban areas and population centres in many countries with cold climates. Nonetheless, due to higher demands and costs associated with expanding DH systems, there is a growing interest in hybrid solutions that integrate several heat sources. Building-integrated Heat Pumps (HPs), with their potential to use renewable energy sources, can greatly support conventional DH systems [1]. They are rapidly spreading and gaining popularity in countries with cold winter climates, such as Germany, Sweden, Denmark, and Austria, due to their decreasing costs and government incentives [1]. Not only are they able to assist to supply the heating demand in cold seasons, but these units also provide a highly efficient source of heating for end-users [2]. This generally results in lower energy bills for building managers and reduces the DH network demand by shifting the load to the electricity network.

Developing a thermal energy model of the building can assist in predicting the indoor temperature while capturing the dynamics of the building —such as inertia and thermal capacity—leading to better-informed decisions. A resistance-capacitance (RC) equivalent

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network model, often categorized as gray-box modelling [3], provides a fairly accurate approximation of a building's dynamic behavior without a need for more computationally intensive simulations [4]. This approach, while simplifying laborious thermodynamic heat transfer calculations, has the advantage of flexible complexity, meaning that the model accuracy is correlated with the assumed number of building parameters. Tarragona et al. provides a review of the available building RC models with different numbers of parameters, and a particle swarm optimization-based algorithm is used for parameter identification of a 4R3C model [10]. In a research work by Plaum et al., a 3R2C model is considered, and a thermal parameter estimation for this model is proposed based on the available building thermal guidelines [5]. A 5R3C RC-model is developed by Golmohamadi et al. for buildings in different climate zones, and the building parameters are identified using a stochastic approach [6].

In environments where multiple energy sources are available, a rigorous control strategy is needed to enable flexible and cost-effective operation. This control can be central, distributed, or hybrid. Model Predictive Control (MPC) functions as a online control system with robust capabilities, addressing prediction errors and model mismatches in operational optimization by incorporating feedback from the network itself [7]. A study by Taylor et al. focuses on economic MPC (eMPC), which aims to minimize the cost, to control 5th generation district heating and a water source HP, along with other auxiliary local heating resources from a building perspective [8]. This controller uses a Mixed-Integer Linear Programming (MILP) rule-based model. Hermansen et al. investigated a simplified, linear MPC proposed to control a heat booster substation in an ultra-low temperature DH system connected to a small HP, a large HP, and thermal storage [9]. The simplified linear model, while providing high computational speed, is not capable of capturing all the complexity of the thermal nonlinear system. An economic evaluation of a hybrid heating system using an MPC algorithm, which includes a HP, thermal energy storage (TES), and photovoltaic (PV) panels, is presented by Tarragona et al. [10]. Additionally, an eMPC algorithm is developed in [6] to adjust heat consumption in response to electricity prices.

In the current literature, integrated operation of HP-DH systems, specifically from the perspective of building dynamics, remains insufficiently investigated. This gap highlights the need for a comprehensive exploration of how HP-DH systems can be effectively managed and optimized when considering the intricate thermal and

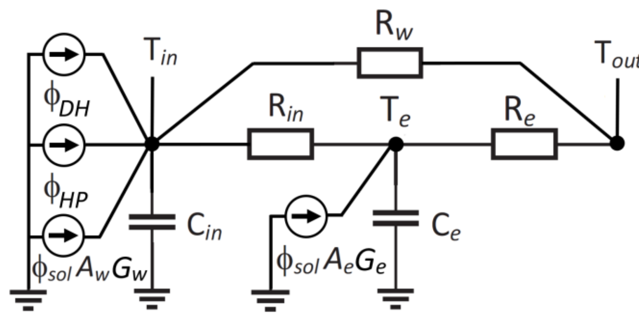

Fig. 1 RC network of the building thermal properties

operational behaviors of buildings. This paper presents a nonlinear-based eMPC control algorithm to optimize a hybrid heating system in a short-term horizon. The objective is to minimize the total cost of the DH-HP hybrid system from the end-user perspective. To build a rigorous model, the dynamics of the building are incorporated into the model with a state-space model based on a RC thermal algorithm.

The rest of the paper is organized as follows: Section 2 explains the resistor-capacitor model as well as the controller system adopted by this paper. Section 3 outlines the simulation results. Section 4 discusses the the authors' interpretation of the results. Finally, Section 5 provides conclusions and suggests future work.

2. MATERIALS AND METHODS

2.1 Building Thermal Modelling

This paper incorporates a 3R2C -three thermal resistances and two thermal capacitances-model applied to a single-family residential building test case study. The RC network representing the building model is depicted in Fig. 1. It is worth mentioning that in our analysis, HP, DH, and solar irradiation on the building's envelope and windows are considered as the sources of heating energy. Conversely, for the sake of simplicity, process energy (including person energy, lighting energy, and appliances energy) and indoor furniture thermal resistance/capacitance are excluded from this study.

The general formulation of the state-space model based on Fig. 1 is expressed in equations (1) and (2). The model was proposed by Wang et al. [11].

$$\begin{aligned}\dot{T}_{in} &= \frac{1}{R_{in}C_{in}}T_e - \left(\frac{1}{R_{in}C_{in}} + \frac{1}{R_wC_{in}}\right)T_{in} + \frac{1}{R_wC_{in}}T_{out} \\ &\quad + \frac{1}{C_{in}}(\phi_{HP} + \phi_{DH}) \\ &\quad + \frac{1}{C_{in}}\phi_{sol}A_wG_w \qquad (1)\end{aligned}$$

$$\dot{T}_e = -\left(\frac{1}{R_{in}C_e} + \frac{1}{R_eC_e}\right)T_e + \frac{1}{R_{in}C_e}T_{in} + \frac{1}{R_eC_e}T_{out} + \frac{A_eG_e}{C_e}\phi_{sol} \quad (2)$$

2.2 *Economic Model Predictive Control for Hybrid Energy System*

MPC is an online control strategy that aims to find the optimal value of an objective function. Given the current input signals and their future forecasts, MPC iteratively calculates and adjusts the control variables over a predefined control horizon, continuously optimizing system performance to meet specified objectives while adhering to constraints. In eMPC, the objective function incorporates economic cost considerations.

Based on the state-space building model mentioned in Section 2.1, the adopted controller has two states, two inputs, and one output which are mentioned in equation (5). The primary goal of the controller in this research is to reduce the building's heat energy consumption and cost by choosing the most efficient resource based on current electricity and heating market prices. The controller parameters of the eMPC are the share of each heating resource over the simulation horizon. The objective function and main constraints of the problem are defined as follows:

$$Obj{:}\ Min. \sum_t C_{El}(P_{HP}(t)) + C_{DH}\left(\frac{\phi_{DH}(t)}{\eta_{DH}}\right) + w_k\varepsilon_k \quad (3)$$

Subject to:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \quad (4)$$

$$x = \begin{bmatrix} T_{in} \\ T_e \end{bmatrix}, u = \begin{bmatrix} \phi_{HP} \\ \phi_{DH} \\ \phi_{sol} \\ T_{out} \end{bmatrix}, y = [T_{in}] \quad (5)$$

$$\begin{cases} A = \begin{bmatrix} -\left(\frac{1}{R_{in}C_{in}} + \frac{1}{R_wC_{in}}\right) & \frac{1}{R_{in}C_{in}} \\ \frac{1}{R_eC_e} & -\left(\frac{1}{R_{in}C_e} + \frac{1}{R_eC_e}\right) \end{bmatrix} \\ B = \begin{bmatrix} \frac{1}{C_{in}} & \frac{1}{C_{in}} & \frac{A_wG_w}{C_{in}} & \frac{1}{R_wC_{in}} \\ 0 & 0 & \frac{A_eG_e}{C_e} & \frac{1}{R_eC_e} \end{bmatrix} \\ C = [1 \quad 0], \qquad D = [0 \quad 0 \quad 0 \quad 0] \end{cases} \quad (6)$$

Table 1. Thermal network parameters

Symbol	Unit	Value	Symbol	Unit	Value
Building Parameters					
C_{in}	$\left[\frac{kWh}{°C}\right]$	2.749	R_{in}	$\left[\frac{°C}{kW}\right]$	2.166
C_e	$\left[\frac{kWh}{°C}\right]$	9.186	R_e	$\left[\frac{°C}{kW}\right]$	21.243
A_w	$[m^2]$	20	G_w	$[-]$	0.5
A_e	$[m^2]$	180	G_e	$[-]$	0.1
R_w	$\left[\frac{°C}{kW}\right]$	41.66			
Heating System Parameters					
ϕ_{HP}^{max}	$[kW]$	15	P_{HP}^{max}	$[kW]$	5
ϕ_{HP}^{min}	$[kW]$	0	P_{HP}^{min}	$[kW]$	0
COP_{HP}	$[-]$	3.5	η_{DH}	$[-]$	0.9

$$P_{HP}(t) = \frac{\phi_{HP}(t)}{COP_{HP}} \quad (7)$$

$$T_{in}^{min} + \varepsilon_k \le T_{in}(t) \le T_{in}^{max} + \varepsilon_k \quad (8)$$

$$P_{HP}^{min} \le P_{HP}(t) \le P_{HP}^{max} \quad (9)$$

$$\phi_{DH}^{min} \le \phi_{DH}(t) \le \phi_{DH}^{max} \quad (10)$$

In equation (3), the first and second terms refer to electricity and DH cost calculation, and the third term refers to the soft constraint on indoor temperature, introduced with a slack variable. The general controller state-space mode, our model components, and state-space coefficients are presented in equations (4), (5) and (6). In equation (6), the coefficient matrices A, B, C, and D are deduced based on the state-space model presented in equations (1) and (2). The electricity power of HP is calculated with equation (7). The constraints regarding the upper and lower limits of indoor temperature, HP electricity, and DH heating power are shown in equations (8), (9), and (10), respectively.

The state-space model and eMPC controller are designed in MATLAB® and Simulink® 2019 environment.

3. RESULTS

3.1 *Simulation data*

To test the eMPC controller presented in this paper, a simulated test case study of a 1-zone, 60-m^2 single-family building located in Malmö, Sweden, is considered. The thermal parameters are calculated based on the guidelines given by Plaum et al. for a medium-weight apartment [5]. Table 1 shows the building and heating

Table 2. electricity and district heating energy prices

Hours	0-10	10-21	21-24
Network status	Off-peak	Peak	Off-peak
Electricity Price (€/kWh) - S1	0.1	0.4	0.1
Electricity Price (€/kWh) - S2	0.2	0.6	0.2
DH Price (€/kWh)	0.1037		

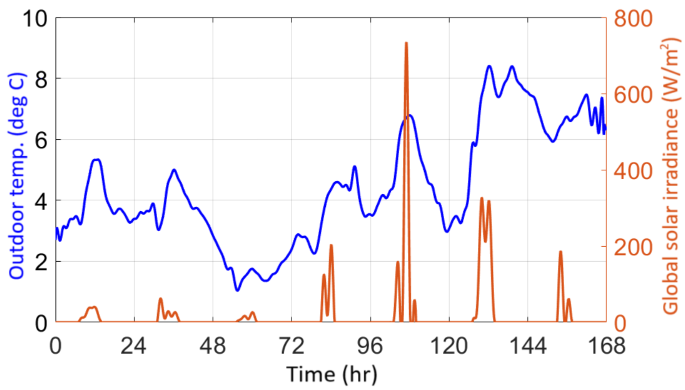

Fig. 2 Meteorological data (ϕ_{sol}, T_{out}) representing the simulation horizon

system parameters and the values that were adapted for the simulation section of this work. Additionally, the meteorological signal data indicating the outdoor temperature and solar irradiance of Malmö, Sweden, are retrieved from an online database [12] and shown in Fig. 2, representing the first week of December 2020. Finally, to monetize the controller cost function, separate electricity prices and DH pricing signals are needed. Based on household energy pricing in south of Sweden [13] two assumed scenarios (S1 and S2) for a time of use-based electricity prices and one DH pricing tariff are considered which are presented in Table 2.

3.2 Numerical Results

To demonstrate the efficiency of the eMPC algorithm, a base case with an on/off controller with a single energy supplier at a time is also considered. This controller operates by switching the supplier on when the temperature drops below the lower threshold and switching it off when the temperature reaches the upper threshold. This will leave us with three different cases:

(C1) On/off controller HP supplier.
(C2) On/off controller DH supplier.
(C3) eMPC controller hybrid (DH-HP) supplier.

The cost and energy consumption of each case with S1 and S2 prices, mentioned in Section 3.1, over one week of simulation is calculated and depicted in Table 3. Additionally, Fig. 3, Fig. 4, and Fig. 5 represent the indoor temperature and heating power of the HP and DH over the simulation horizon for case (C1), case (C3) with S1 prices, and case (C3) with S2 prices, respectively. The graphs representing case (C2) are not added due to similar attributes with case (C1) and to avoid repetition.

Table 3. Simulation results on different controllers with S1 and S2 scenario prices

Controller case	(C1) On/off controller (HP)		(C2) On/off controller (DH)		(C3) eMPC controller (hybrid)	
Price Scenario	S1	S2	S1	S2	S1	S2
Electricity consumption (kWh)	79.5		0		70	19
DH heat consumption (kWh)	0		313.33		0	35
Simulation Cost (€)	19.5	31.32	32.49		13.75	18.57

4. DISCUSSION

When analysing the simulation results and the behavior of the eMPC algorithm compared to the basic on/off controller, some observations are made. Overall, Table 3 indicate that the eMPC system results in a

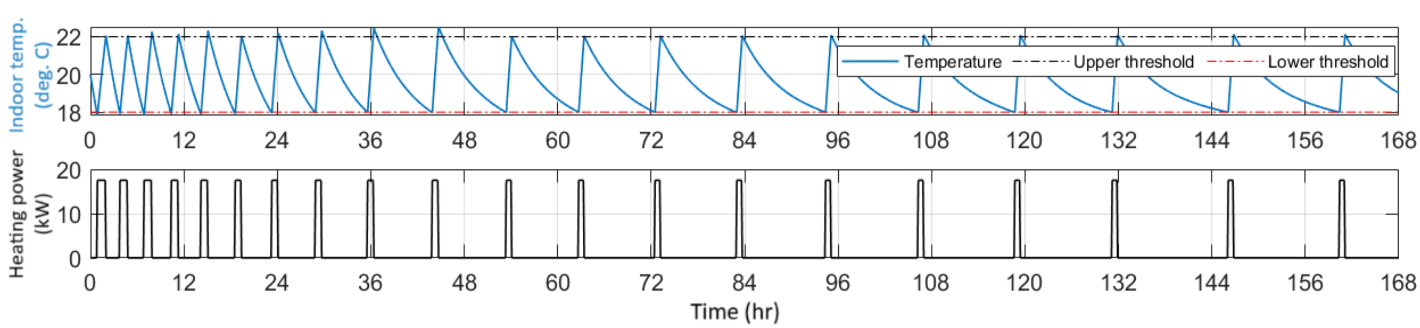

Fig. 3 Simulation results of the case C1 - on/off controller with heat pump as the supplier for both S1 /S2 prices

substantial cost saving from 29% up to 57% in S1, and nearly 40% under S2 prices over the simulation horizon in winter days. This saving in consumed energy and cost shows the effectiveness of the controller and hybrid DH-HP solution. It should be noted that we are here not taking into account the additional investment and operational costs related to the HP.

One apparent observation on the graphs is that the number of switches of the heat supplier in the eMPC controller is notably higher than that in the basic controller. The shortest interval between two on switches in the eMPC and the on/off controller were 0.8 hrs and 1.8 hrs, respectively.

Additionally, it was noted that the introduced slack variable makes the eMPC more forgiving to the changes in temperature, retaining it between 17.2 and 20.5. The on/off controller, however, has a stricter range of 17.9 to 22.0 degrees. Thus, this version of the eMPC probably will provide lower comfort for the residents.

Another remark is the price responsiveness. It can be observed in Fig. 4 and Fig.5 when compared to Fig. 3 that unlike the on/off controller, the eMPC reacts to the changes in electricity prices. The system tends to increase the temperature just before (or sometimes at the beginning of) peak hours to use the building as heat storage during more expensive hours and use the heating system less frequently. This has resulted in cost savings during peak hours.

Finally, another noticed element was the effect of price schemes on the hybrid energy system. Comparing Fig. 4 and Fig. 5, it is observed that the eMPC algorithm with S1 prices only tends to employ HP heating systems. Under the S2 pricing scheme, nevertheless, the DH prices become competitive in peak electricity hours, and the system starts using both heating systems throughout the simulation horizon.

5. CONCLUSIONS AND FUTURE WORK

In this paper, the application of an economic Model Predictive Controller (eMPC) system is simulated on a hybrid energy system of a single-family building. A base case of an on/off controller is considered to compare the results where possible. The heating system incorporates a building-integrated HP and central DH system. To capture the dynamic thermal attributes of the building, a state-space model with a resistor-capacitance network of the building is adopted. Some of the key findings of this research are:

- The proposed controller resulted in savings up to 57% on energy bills.
- The eMPC demonstrates price responsiveness effects, meaning that the energy consumption

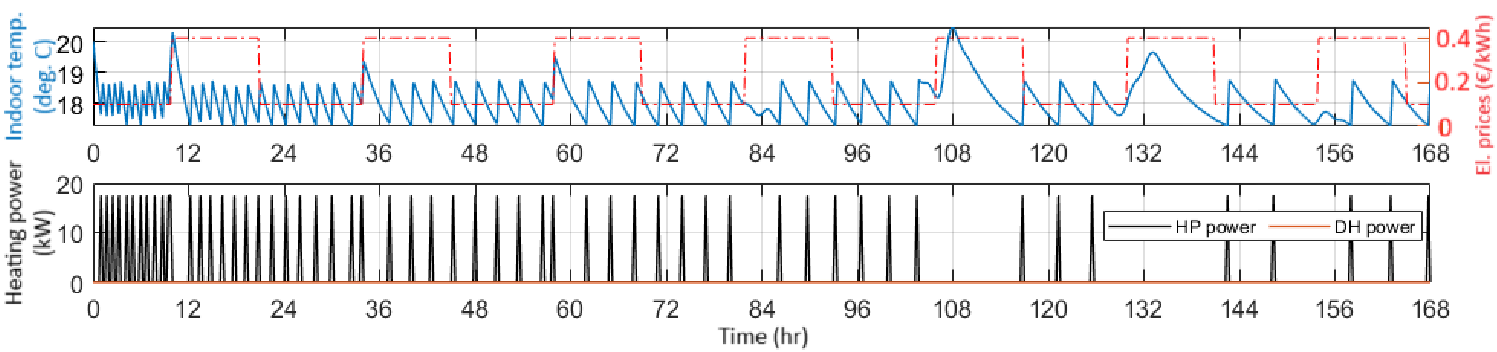

Fig. 4 Simulation results of the case C3 - eMPC controller with S1 prices

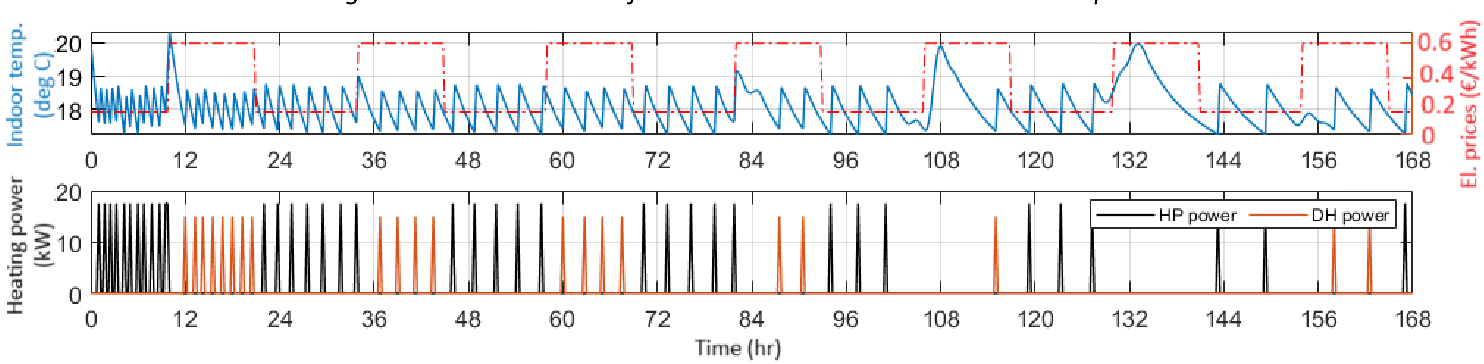

Fig. 5 Simulation results of the case C3 - eMPC controller with S2 prices

pattern is changed when prices are altered during off-peak and peak hours.
- While in S1 scenario HP is more cost-effective and the entire heat demand is supplied by this local unit, under the S2 electricity pricing, the DH prices become competitive, and they can support HP in a hybrid mode.

For future developments of this work, additional objectives could be taken into account, for instance, environmental concerns like CO_2 emissions and social effects like comfort level. In addition, further case studies with different applications, such as commercial buildings, under different weather conditions could be investigated to better understand the system behavior under changing circumstances.

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