Model Predictive Control of PEMFC Air Supply System Based on Kalman State Estimation of Unmeasurable Parameters

Haowen Shen¹, Fengxiang Chen^{1*}

1 School of Automotive Studies, Tongji University, Shanghai 201804, China (Corresponding Author: fxchen_qjy@hotmail.com)

ABSTRACT

The air supply subsystem is critical for proton exchange membrane fuel cells, determining the output performance of the stack. By controlling the speed of the air compressor and the opening of the back pressure valve, it is possible to achieve appropriate regulation of air supply pressure and mass flow rate. However, this process presents challenges due to disturbances and the coupling of multiple variables. This study employs the model predictive control algorithm to address these issues because of its strong decoupling capabilities and robustness. Initially, an M‐sequence is designed to identify the system and obtain the predictive model for the MPC. Based on feedback output, a Kalman filter is then used to estimate the optimal unmeasurable state information. Subsequently, the MPC controller is designed to obtain the optimal control output under various constraints. Finally, by using a traditional PID controller as a control group, the performance of the proposed MPC controller based on Kalman state estimation is analyzed under current step change conditions.

Keywords: fuel cell, model predictive control, Kalman estimation

1. INTRODUCTION

Proton exchange membrane fuel cells(PEMFCs) have been used in vehicle applications in recent years because of its advantages of low emission, high efficiency and fast refueling [1]. As one type of battery, fuel cells generate electricity through chemical reactions between hydrogen and oxygen [2]. For the purpose of improving the output power and increasing the service life of stack, fuel cell engines need to keep the operating parameters such as stack temperature, gas humidity, mass flow rate and pressure in the appropriate state. The air supply system is mainly used to provide the necessary reaction gas for the cathode. Two key parameters of gas supply, namely air mass flow rate and pressure, are essential for the efficient and healthy operation of fuel cell systems [3,4]. As a typical multi-input-multi-output coupling system [5], the high‐pressure fuel cell air supply system has nonlinearity and uncertainty. Designing corresponding control algorithms to ensure its key parameters work under predetermined conditions is necessary.

Many research studies have focused on the regulation of the air supply system. Model predictive control(MPC) algorithm has also been used in the parameter control of fuel cells because of its excellent control performance and relatively simple design process. Abdullah et al. [6] used constrained MPC to optimize oxygen excess ratio by solving the voltage of air compressor. Hahn et al. [7] designed MPC to solve the optimal operating conditions with the goal of minimizing the operating power. Hu et al. [8] designed MPC to control oxygen excess ratio by fusing multiple equilibrium point linearization models. There are also some studies combining model predictive control with other control methods to achieve good performances [9-11].

In this paper, the model prediction model is established by means of system identification, which is used to solve the MIMO pressure and flow coupling cooperative control problem. To solve the problem of state estimation of unmeasured state variables, this paper proposes to combine Kalman filter with model predictive control to solve the closed‐loop problem. The introduced noise modeling can effectively improve the accuracy of the identification model, thus making the control algorithm performing better.

2. METHODOLOGY

2.1 Model of the air supply system

The main components of air supply system for PEMFC include air filter, air compressor, intercooler,

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humidifier, back pressure valve, etc. In this paper, the coupling control of the pressure and flow of the stack is realized by controlling the speed of the air compressor and the opening degree of the back pressure valve. The system is shown in Fig.1. It is a typical fuel cell air supply system structure

Based on previous work [12], a global representation

Where $x1$, $x2$, $x3$, $x4$ are state variables. Taking the previously designed M-sequence as input,

of air supply system was established. In this paper, the M‐sequence identification method was used to identify a 150kw high‐pressure fuel cell air supply system. We choosed a steady‐state point for the following work. For 450A‐current, the steady‐state mass flow rate is 153.19g/s and the pressure is 241.1kpa according to the experimental results, corresponding to the air compressor speed of 64711r/min and the backpressure valve opening degree of 14.54%. First-order inertia link of the following form is selected:

$$
\begin{bmatrix} \dot{m} \\ p \end{bmatrix} = \begin{bmatrix} \frac{k_{11}}{T_{11}s + 1} & \frac{k_{12}}{T_{12}s + 1} \\ \frac{k_{21}}{T_{21}s + 1} & \frac{k_{22}}{T_{22}s + 1} \end{bmatrix} \begin{bmatrix} n \\ \theta \end{bmatrix}
$$
 (1)

Where k_{11} , k_{12} , k_{21} , k_{22} , T_{11} , T_{12} , T_{21} , T_{22} are the parameters of the mapping relationship; \dot{m} , p are mass flow rate(g.s-1) and pressure(kpa); n, θ are the air compressor speed(r/min) and the backpressure valve opening degree(%),so that the linear model of the equilibrium point is established. It is transformed into the state space equation form to further design the model predictive controller. Finally, the following results are obtained In the form of four state variables, two inputs and two outputs:

the output of the equilibrium point identification model and the actual model were compared, and the agreement of pressure and mass flow rate is 95% and 93%, respectively. Therefore, the linear model near the equilibrium point above is reasonable.

This result will be used in the subsequent design.

2.2 Model Predictive Control with state Estimation

Model predictive control has excellent dynamic control performance and multi‐variable control ability. The principle is that at each sampling time, the objective function is established through the prediction model, and the quadratic programming problem is subsequently solved online, and the first element of the obtained control sequence is applied to the controlled plant. At the next sampling time, it needs to use the new state value as the initial condition of the prediction system at this time and solve it again. If the state variables are not completely measurable, a state estimator needs to be designed.

The state variables in Eq. (2) are the intermediate variables of the identification model, which cannot be directly transmitted to the model predictive controller by output feedback in the real system. In order to solve the

estimation problem of unmeasurable state variables in air supply system, this paper uses Kalman filter to process the measured value of the actual system output to obtain the state quantity. Fig.2 is the control structure. According to the demand current, the reference pressure and the reference flow of the model predictive controller are obtained by looking up the table online according to the experimental calibration table.

Fig. 2 The proposed control structure

2.2.1 Kalman filter

According to Eq. (2), $x \cdot y \cdot u$ are defined as follows.

$$
\begin{cases}\n x = [x1 \ x2 \ x3 \ x4]^T \\
 u = [n \ \theta]^T \\
 y = [m \ p]^T\n\end{cases}
$$
\n(3)

Eq. (2) is further transformed into a discrete state space equation, and process noise and measurement noise are added to the model, which is expressed as follows:

$$
\begin{cases} x_{k+1} = A_m x_k + B_m u_k + w_k \\ y_k = C_m x_k + v_k \end{cases} \tag{4}
$$

Here, we assume that the process noise w_k and measurement noise v_k follow the Gaussian distribution with the expectation of 0, and the process noise and measurement noise are uncorrelated. A_m , B_m , C_m are the model system matrix, control matrix, and output matrix respectively. $k, k+1$ are $k, k+1$ sampling moments, respectively.

Kalman filter needs to obtain the output of the actual air supply system model at the current time and the input of the actual system at the previous time to solve the state variables at the current time. Usually, Kalman filter is divided into two parts, namely, the state prediction and the state updating.

The state prediction part is:

$$
\hat{x}_k^- = A_m \hat{x}_{k-1}^+ + B_m u_{k-1} \tag{5}
$$

 $P_k^- = A_m P_{k-1}^+ A_m^T + Q$ (6)

And the state updating part:

$$
K_k = \frac{P_k^- C_m^T}{C_m P_k^- C_m^T + R}
$$
(7)

$$
P_k^+ = (I - K_k C_m) P_k^- \tag{8}
$$

$$
\hat{x}_k^+ = \hat{x}_k^- + K_k(y_k - C_m \hat{x}_k^-)
$$
\n(9)

where \hat{x}_k^- stands for the priori estimated state and \hat{x}_{k}^{+} is the posterior estimated state. Because the posterior information y_k is added to modify it, the estimated values are closer to the real state variables. K_k is the Kalman gain matrix updated at each sampling moment, P_k^+ stands for the posterior error covariance matrix of \hat{x}_k^+ , P_k^- represents priori error covariance matrix of \hat{x}_k , R is the variance of measuring noise v_k , Q is the variance of process noise w_k . We assume that P_k^+ , P_k^- , R and Q are diagonal matrices of the corresponding covariance dimensions. The variance of the error will be gradually reduced through the feedback of the real model measurements.

2.2.2 Model predictive control

After obtaining the state estimate \hat{x}_k^+ , It can be used as the initial condition of the prediction model to design the mpc controller through the mathematical model of the system. Here, we solve the problem with an incremental form of MPC.

Define
$$
\Delta u_k
$$
, Δx_k :
\n
$$
\begin{cases}\n\Delta u_k = u_k - u_{k-1} \\
\Delta x_k = x_k - x_{k-1}\n\end{cases}
$$
\n(10)

Define the state quantity
$$
\bar{x}_k
$$
 as follows:
\n
$$
\bar{x}_k = [A x_k \quad v_k]^T
$$
\n(11)

Rearrange the state space equation in incremental form, which is defined as follows:

$$
\begin{cases} \overline{x}_{k+1} = A\overline{x}_k + B\Delta u_k \\ y_k = C\overline{x}_k \end{cases}
$$
 (12)

Let the current sampling time be k , the size of the prediction window be N_p sampling intervals, and define N_n as the prediction time domain. In the prediction time domain, from sampling time k , there are manipulated variables with N_c sampling intervals, and N_c is defined as the control time domain.

Define
$$
Y_k
$$
 and ΔU_k :
\n
$$
\begin{cases}\nY_k = [Y_{k+1} \quad Y_{k+2} \quad \cdots \quad Y_{k+N_p}]^T \\
\Delta U_k = [\Delta u_k \quad \Delta u_{k+1} \quad \cdots \quad \Delta u_{k+N_c-1}]^T\n\end{cases}
$$
\n(13)

Where y_{k+1} ... y_{k+N_p} stand for system outputs of $k+1... k+N_p$ steps, $\Delta u_k... \Delta u_{k+N_c-1}$ stand for system input changes of $k... k + N_c - 1$ steps.

Eq. (12) is used to predict N_p steps, and the output equation can be obtained by arranging it into a matrix expression as follows:

$$
\begin{cases}\n & Y_k = H\bar{x}_k + F\Delta U_k \\
 & G A^2 \\
 G A^3 \\
 & \dots \\
 G A^{N_p}\n\end{cases}\nF =\n\begin{bmatrix}\n & CB & 0 & \dots & 0 \\
 & CAB & CB & \dots & 0 \\
 & C A^2B & CAB & \dots & 0 \\
 & & \dots & \dots & \dots \\
 & & G A^{N_p-1}B & G A^{N_p-2}B & \dots & G A^{N_p-N_C}B\n\end{bmatrix}
$$
\n(14)

Where \bar{x}_k are known values calculated from the estimated values \hat{x}_{k}^{+} , \hat{x}_{k-1}^{+} and output feedback values \bar{y}_{kout} from fuel cell model:

$$
\bar{x}_k = [\hat{x}_k^+ - \hat{x}_{k-1}^+ \ \bar{y}_{kout}] \tag{15}
$$

We define reference values as R . Here, we smooth the reference track, that is, set the reference track to smoothly transition from the output of the previous moment to the set value according to the predicted step size. The objective function *can be divided into two* parts, E and W are the weights to be determined, and the purpose of predictive control is to solve the manipulated variables to minimize the value of the objective function:

$$
J = (Y_k - R)^T E (Y_k - R) + \Delta U_k^T W \Delta U_k \tag{16}
$$

Combining Eq.(14) and Eq.(15), the optimization of the objective function is transformed into the quadratic programming problem with variable ΔU_k . We consider the constraints of the actual air supply system. The actuator air compressor and back pressure valve have working range and change rate limit. The following formula should be added to the solution:

$$
\begin{cases} U_k = [u_k \quad u_{k+1} \quad \dots \quad u_{k+N_c-1}]^T \\ \text{s.t. } U_{\text{min}} \le U_k \le U_{\text{max}} \\ \Delta U_{\text{min}} \le \Delta U_k \le \Delta U_{\text{max}} \end{cases} \tag{17}
$$

Where U_{min} , U_{max} stand for the speed limit of the air compressor and the opening limit of the back pressure valve, ΔU_{min} , ΔU_{max} stand for the maximum change range within the step.

3. RESULTS AND DISCUSSIONS

The key parameters of model predictive controller include control interval, prediction time domain, control time domain, error weight and control weight. The choice of the control interval is directly related to the update frequency of the control signal. When the control interval is set to be small, the control system will have a high real-time response ability, but too frequent control signal update may exceed the controller's computational processing ability, and then affect the stability and performance of the whole system. Enlarging the prediction time domain and the control time domain means more comprehensive prediction of future changes. However, it will also increase the complexity and time required to solve the problem, posing a challenge to the application scenarios with high real‐time

requirements. As diagonal matrices, the error weights and control weights play a role in balancing the importance of error and control terms. Increasing the control weight appropriately can effectively reduce the fluctuation of the control signal and make the system output more stable. However, this needs to ensure system performance while avoiding excessive smoothing of manipulated variables at the expense of system response speed and adaptability.

The performance of the designed controller is tested by simulation. We first select the current step test in fig.3 near the equilibrium point when the initial state is steady.

Fig. 3 Current condition near the equilibrium point

The pressure and flow rate reference values are obtained from the input current look‐up table, which are further processed based on the change rate limit of the actual system. The control interval is 0.01s, the prediction time domain is 20, the control time domain is 10, the error weight is 10000, and the weight of the control change rate is 10. Compared with the feedforward‐PID algorithm[13] under the same test conditions, the tracking effect is obtained as follows.

Fig. 5 Pressure control results near the equilibrium point

It can be seen that MPC algorithm is obviously better than PID in the range near the equilibrium point, and there is no steady‐state error. Calculating the MSE of both algorithms with reference values, the MPC algorithm is 0.498 in terms of flow rate, which is 32.7% lower than PID, and 0.335 in terms of pressure, which is 59.3% lower than PID.

Then, considering that the equilibrium point model may have errors in the entire nonlinear working region, we verify the control effect in the entire current working range to test the robustness of the algorithm against the model errors. Based on the unstarted state of the fuel cell system, the current step input in fig.6 is provided from zero time, so the starting phase and step dynamic response performance of the fuel cell system can be observed in fig.7 and fig.8.

In the start‐up phase, MPC can complete the start‐ up in 4s, which has a faster response speed than PID.

In the current step condition, MPC can quickly track the reference value. Furthermore, it can be seen from the enlarged figure that MPC has smaller pressure flow fluctuation and shorter time to reach steady state than PID.

Fig. 7 Mass flow rate control results in full range

The MSE values of the two control algorithms compared with the reference values were calculated. For the pressure curve, the value of MPC is 1.440, 57.2% lower than PID. For the flow rate curve, the value of MPC is 1.968, 39.0% lower than PID.

The above simulation results show that the model predictive controller based on Kalman state estimation designed in this paper has a excellent control effect. For the current region far from the equilibrium point, although the control effect is slightly worse than that near the equilibrium point, it can still maintain zero steady‐state error and fast response speed. In the future, multiple equilibrium point models can be considered to design MPC, which may achieve better control over the entire operating range.

4. CONCLUSIONS

In this paper, the combination of Kalman filter and model predictive control is proposed for the cooperative control of pressure and flow in air supply systems. Firstly, the physical model of the air supply system is established, and the M‐sequence is selected to identify the model. Afterwards, the noise model is introduced , so Kalman filter can estimate the unmeasurable state variables obtained from the identification. Furthermore, the manipulated variables of air compressor speed and backpressure valve opening can be solved by model predictive control. Simulation results show that the proposed method outperforms the PID method. In addition, the controller parameters are discussed, and the performance of the controller can be intuitively balanced by adjusting the key parameters appropriately.

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