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A semi-analytical model for pressure transient analysis of partially penetrating inclined fracture networks for multi-stage fractured horizontal well

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ABSTRACT

There are some partially penetrating inclined fractures (PPIFs) due to stress interference between fracture networks in multi-stage fractured horizontal wells (MFHWs). It would take tremendous computation time to make flow behavior analysis and well production optimization in numerical simulators. Therefore, it is an extremely urgent task to establish efficient and rigorous models of PPIF networks. In this paper, triangular elements were introduced to characterize the geometry and intersection of PPIFs. Control volume method is adopted to model the fracture flow numerically and the star-delta transformation is used to solve the interconnected fractures more accurately. The fluid flow in the matrix is analytical modeled by source function. The transient flow between PPIFs and matrix is coupled on the surface of fractures. By introduce spatial triangle numerical integral technology in source function calculation, the model efficiency can be greatly increased. Finally, the pressure transient analysis (PTA) of PPIFs can be modelled with high accuracy and less calculation time. With the aid of proposed PPIFs model, the transient pressure behaviors of different MFHWs were analyzed. The hydraulic and natural fractures in these MFHWs have different inclinations, trends, and heights. The flow regimes of PPIFs with finite conductivity are analyzed in detail. Flow regimes analysis indicates that eight flow regimes can be observed, including the wellbore storage flow, fracture linear flow, bilinear flow, formation linear flow, early radial flow, compound linear flow, pseudo-radial flow, and boundary-dominated flow. The results indicate that the formation linear flow and early radial flow would be influenced by the geometry of nature fractures, which is quite different from that of vertical penetration fractures. The introduction of numerical integral method greatly reduces the calculation time (less than 2 minutes) with high accuracy, which make it possible to analysis the fracture parameters more efficiently. The result of typical well test curves analyses shows that the duration of flow regimes will be influenced the parameters of fractures. When the penetration ratio (> 0.6) or the incline angle (>60°) is large enough, the bilinear flow will last longer, and the early radial flow will be absent.

Keywords: Fracture networks, Pressure-transient behavior, Finite conductivity, Semi-analytical model, Source function

NONMENCLATURE

Abbreviations	
PPIFs	Partially Penetrating Inclined Fractures
MFHWs	Multi-stage Fractured Horizontal Wells
Symbols	
β	Unit-conversion factor, 0.0853
$A_{i,k}$	Contact area of adjacent elements, m^2
p	Pressure, MPa
$\vec{c}_{i,k}$	Flow vector from element I to k, dimensionless
$\vec{n}_{i,k}$	Unit vector perpendicular to $A_{i,k}$, dimensionless
q_{mi}	Production from matrix to element I, m^3/d
h_{D}	Dimensionless formation thickness
F_{cD}	Dimensionless hydraulic fracture conductivity
f_{cD}	Dimensionless nature fracture conductivity
$S_{\Delta i}$	Dimensionless area of element i
C_{wD}	Dimensionless wellbore-storage coefficient
θ_{F}	Incline angle of hydraulic fracture, degree
θ_{f}	Incline angle of nature fracture, degree
h_{FD}	Dimensionless height of hydraulic fracture
h_{fD}	Dimensionless height of nature fracture

1. INTRODUCTION

Advancement of MFHWs technology have enabled oil producers to obtain production from unconventional tight shale reservoir economically, the EIA report indicates the production of tight oil has accounted for approximately 68% of the oil production in US [1-2]. Drilling schemes such as infill drilling and well pad were used to increase the density of fractures to obtain higher production. However, with tighter fracture spacing, the risk of fracture connection and interface between hydraulic fractures and nature fractures will significantly increase, which could result in complex fracture networks with different inclined directions, fracture height and fracture geometry [3-4].

Numerical simulation is a useful tool to analysis the performance of fractured well. However, with the increase of fracture complexity, the number of reservoir grids will be enormous, tremendous computation time would be required [5-6]. It would be time costing to interpret the parameters of fractures networks and reservoirs based on the numerical model.

Pressure-transient analysis (PTA) method has become an efficiency reservoir engineering tool for Inverting fracture and reservoir properties [7]. Source function theory is one of the key theories of PTA for MFHWs, which has an obvious advantage in flexibly characterizing the fracture geometry by developing different line and surface source function [8-10]. With the deepening understanding of fracture propagation, various analytical or semi-analytical model have been proposed to analysis the pressure transient behaviors of PPIFs. Cinco-Ley derived the source function, developed an analytical solution for inclined fractures. In this model, the infinite conductivity fully and partially penetrating fractures were considered [11]. Al Rbeawi combined the source function and superposition principle, studied the pressure dynamic of MFHW with inclined fractures. A serious of typical curve of MFHWs with infinite conductivity inclined fractures were obtained based on their work [12]. To character the conductivity and geometry of fractures, discrete fracture network method was introduced. The flux and geometry can be considered by changing the parameters of discrete elements [13]. This method makes it easier to character the geometry of fractures and is a better way to analysis the partially penetrating inclined fractures.

Even though various models have been proposed, most of these works focus on the single fractures and ignore the scenario that inclined fractures could intersect with hydraulic and nature fractures. There are two main problems to analysis this scenario, accurate characterization of PPIF networks and fast algorithm for source functions. In this paper, the triangular elements were introduced to characterize the geometry and intersection of PPIF networks, and the numerical integral technology was used to calculate the source function effectively, which make it more efficient to model the flow in PPIFs, and can analysis the flow regimes of PPIFs quickly.

2. PHYSICAL MODEL AND ASSUMPTIONS

As shown in Fig 1, the fractured well connect with the PPIF networks, in which the fractures have different inclined directions, fracture height and fracture geometry.

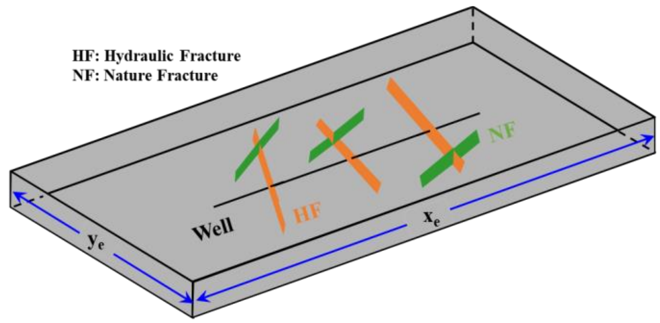

Fig. 1 Schematic diagram of the multi-stage fractured horizontal wells with partially penetrating inclined fracture networks

The assumptions of this model are as follows:

(1) This reservoir is a box-shaped reservoir, with impermeable boundaries.

(2) The matrix system is homogeneous and isotropy.

(3) The fracture is finite conductivity, and the properties of the same fracture are the same.

(4) Oil flow from matrix to fracture, and then from fracture to wellbore, ignore the oil flow from matrix to wellbore.

(5) The fluid in the reservoir is oil, ignore the influence of capillary.

(6) The wellbore-storage effect is considered in the model.

3. MATHEMATICAL MODEL AND SOLUTION

3.1 Fracture flow model

The fracture surface is discrete into triangular elements. The oil flow in fractures can be obtained by calculating the transmissibility among different elements.

The mass conservation equations of the i[th] element can be written as follows.

$$-\int_{\partial\Omega_i} \rho v ds + q = \frac{\partial(\rho V_i \phi)}{\partial t} \tag{1}$$

Where, the velocity between the i[th] element and adjacent k[th] element can be written as follows:

$$v_{i,k}=\int_{\Gamma_{ik}}\vec{v}\cdot\vec{n}ds\approx A_{i,k}\vec{v}\left(\vec{x}_{i,k}\right)\cdot\vec{n}_{i,k}$$
$$=-\frac{A_{i,k}}{\mu}\frac{\left(p_i-\pi_{i,k}\right)\vec{c}_{i,k}}{\left|\vec{c}_{i,k}\right|^2}\cdot\vec{n}_{i,k} \quad (2)$$

Substitute Eq. 2 into Eq. 1 and convert into finite-difference format.

$$\sum_{k=1}^{n_i}\frac{\beta A_{i,k}k_i}{\mu B}\frac{\left(p_i^{n+1}-\pi_{i,k}^{n+1}\right)\vec{c}_{i,k}}{\left|\vec{c}_{i,k}\right|^2}\cdot\vec{n}_{i,k}+q_{mi}=\frac{V_i\phi_f C_{tf}}{B}\frac{p_i^{n-1}-p_i^n}{\Delta t} \quad (3)$$

The dimensionless format can be obtained.

$$\sum_{k=1}^{n_i}\frac{F_{cD}A_{Di,k}\vec{c}_{Di,k}\vec{n}_{i,k}\left(p_{Dk}-p_{Di}\right)}{2\pi h_D\left|\vec{c}_{Di,k}\right|^2}+q_{miD}^n=\frac{C_sA_{Di}w_D}{2\pi h_D\Delta t_D}\left(p_{Di}^n-p_{Di}^{n-1}\right) \quad (4)$$

The transmissibility between two adjacent can be written as follows.

$$T_{ik}=\frac{T_{i,k}\cdot T_{k,i}}{T_{i,k}+T_{i,k}} \quad (5)$$

Where,

$$T_{i,k}=\frac{F_{cD}A_{Di,k}\vec{c}_{Di,k}\vec{n}_{i,k}}{2\pi h_D\left|\vec{c}_{Di,k}\right|^2},T_{k,i}=\frac{F_{cD}A_{Di,k}\vec{c}_{Dk,i}\vec{n}_{k,i}}{2\pi h_D\left|\vec{c}_{Dk,i}\right|^2}$$

Especially, for the elements between the interconnected fractures, the transmissibility can be obtained based on the Star-Delta transformation [14].

$$T_{ik}=\frac{T_{i,k}\cdot T_{k,i}}{\sum_{m=1}^{ne}T_{i,m}} \quad (6)$$

The general flow equation of triangular elements can be obtained.

$$\sum_{k=1}^{N_i}T_{ik}p_{Dk}^n-\left(\sum_{k=1}^{N_i}T_{ik}+\alpha_{ik}\right)p_{Di}^n+\left(q_i^n-q_w^n\right)=-\alpha_{ik}p_{Di}^{n-1} \quad (7)$$

The flow equations matrix for all N_f triangular elements can be obtained.

$$\left[T,-I_{N_f},o,a\right]\cdot\begin{bmatrix}p_{fD}\\q_{fD}\\p_{fwD}\\q_{fwD}\end{bmatrix}=R_1 \quad (8)$$

3.2 Reservoir flow model

The analytical solution of point source in a bounded reservoir can be written as.

$$p_{D0}=\frac{2\pi q_D}{x_{eD}y_{eD}}\cdot\left[1+2\sum_{m=1}^{\infty}\exp\left(-\frac{m^2\pi^2t_D}{x_{eD}^2}\right)\cos\frac{m\pi x_{D0}}{x_{eD}}\cos\frac{m\pi x_D}{x_{eD}}\right]\cdot\left[1+2\sum_{m=1}^{\infty}\exp\left(-\frac{m^2\pi^2t_D}{y_{eD}^2}\right)\cos\frac{m\pi y_{D0}}{y_{eD}}\cos\frac{m\pi y_D}{y_{eD}}\right]\cdot\left[1+2\sum_{m=1}^{\infty}\exp\left(-\frac{m^2\pi^2t_D}{h_D^2}\right)\cos\frac{m\pi z_{D0}}{h_D}\cos\frac{m\pi z_D}{h_D}\right] \quad (9)$$

According to the superposition principle, the plane source of triangular elements can be obtained by Integration along the element surface, which is time-consuming. Therefore, the numerical integral theorem for spatial triangle is introduced. Based on the surface integral of a scalar field, the integral of spatial triangle can be rearranged as follows. By using this numerical method, the calculation time decrease greatly from about 2 hours to no more than 1 minutes, which improve the computational efficiency greatly.

$$\iint_S f(x,y,z)\mathrm{d}S=\iint_D f(x,y,z(x,y))\sqrt{1+z_x^2+z_y^2}dxdy\approx S_{\Delta ABC}\sum_{i=1}^{N}f(x_{1i},y_{2i},z_{3i}) \quad (10)$$

The pressure response of j[th] element caused by the fracture system can be calculated by summing the pressure response from all the elements.

$$p_{Dj}^n=\frac{2\pi}{x_{eD}y_{eD}}\sum_{i=1}^{N_f}\sum_{s=1}^{N}\sum_{k=1}^{n}S_{\Delta i}q_{Di}^k\int_{t_D^{k-1}}^{t_D^k}\cdot\left[1+2\sum_{m=1}^{\infty}\exp\left(-\frac{m^2\pi^2(t_D-\tau)}{x_{eD}^2}\right)\cos\frac{m\pi x_{Dj}}{x_{eD}}\cos\frac{m\pi x_{Dis}}{x_{eD}}\right]\cdot\left[1+2\sum_{m=1}^{\infty}\exp\left(-\frac{m^2\pi^2(t_D-\tau)}{y_{eD}^2}\right)\cos\frac{m\pi y_{Dj}}{y_{eD}}\cos\frac{m\pi y_{Dis}}{y_{eD}}\right]\cdot\left[1+2\sum_{m=1}^{\infty}\exp\left(-\frac{m^2\pi^2(t_D-\tau)}{h_D^2}\right)\cos\frac{m\pi z_{Dj}}{h_D}\cos\frac{m\pi z_{Dis}}{h_D}\right]d\tau \quad (11)$$

Define that

$$G_{i,j}^{n,k}=\frac{2\pi}{x_{eD}y_{eD}}\sum_{s=1}^{N}S_{\Delta i}\int_{t_D^{k-1}}^{t_D^k}\left[1+2\sum_{m=1}^{\infty}\exp\left(-\frac{m^2\pi^2(t_D-\tau)}{x_{eD}^2}\right)\cos\frac{m\pi x_{Dj}}{x_{eD}}\cos\frac{m\pi x_{Dis}}{x_{eD}}\right]\cdot\left[1+2\sum_{m=1}^{\infty}\exp\left(-\frac{m^2\pi^2(t_D-\tau)}{y_{eD}^2}\right)\cos\frac{m\pi y_{Dj}}{y_{eD}}\cos\frac{m\pi y_{Dis}}{y_{eD}}\right]\cdot\left[1+2\sum_{m=1}^{\infty}\exp\left(-\frac{m^2\pi^2(t_D-\tau)}{h_D^2}\right)\cos\frac{m\pi z_{Dj}}{h_D}\cos\frac{m\pi z_{Dis}}{h_D}\right]d\tau \quad (12)$$

The pressure drop general equation for each element can be concluded as follows:

$$p_{\mathrm{Dj}}^{n}=\sum_{i=1}^{N_{\mathrm{f}}}\sum_{k=1}^{n}q_{\mathrm{Di}}^{k}G_{i,j}^{n,k}=q_{\mathrm{Di}}^{k}\sum_{i=1}^{N_{\mathrm{f}}}G_{i,j}^{n,k}+\sum_{i=1}^{N_{\mathrm{f}}}\sum_{k=1}^{n-1}q_{\mathrm{Di}}^{k}G_{i,j}^{n,k} \quad (13)$$

These matrix-flow equations can be arranged into a matrix format, which is given as follows:

$$B\cdot\begin{bmatrix}p_{\mathrm{fD}}\\ q_{\mathrm{fD}}\\ p_{\mathrm{fwD}}\\ q_{\mathrm{fwD}}\end{bmatrix}=R_2 \quad (14)$$

3.3 *Solution Methodology*

Besides the equation of fracture flow and reservoir flow, the equation for the wellbore and well storage can be considered as follows.

$$p_{\mathrm{fwD}}^{n}-p_{\mathrm{fD}}^{n}=\frac{w_{\mathrm{D}}q_{\mathrm{fwD}}^{n}\ln(r_{\mathrm{eqD}}/r_{\mathrm{wD}})}{F_{\mathrm{cD}}} \quad (15)$$

$$1-\frac{\gamma C_{wD}}{\Delta t_D^n}\left(p_{\mathrm{wD}}^{n}-p_{\mathrm{wD}}^{n-1}\right)=\sum_{i=1}^{M}q_{\mathrm{fwDi}}^{n} \quad (16)$$

$$p_{\mathrm{fwDi}}^{n}=p_{\mathrm{fwD(i+1)}}^{n},\, i\in(2\cdots M) \quad (17)$$

These equations related to the wellbore can be arranged into a matrix format.

$$C\begin{bmatrix}p_{\mathrm{fD}}\\ q_{\mathrm{fD}}\\ p_{\mathrm{fwD}}\\ q_{\mathrm{fwD}}\end{bmatrix}=R_3 \quad (18)$$

The total matrix of MFHW with PPIFs can be obtained by simultaneous equations 9, 14 and 18.

$$\begin{bmatrix}A\\ B\\ C\end{bmatrix}\cdot\begin{bmatrix}p_{\mathrm{fD}}\\ q_{\mathrm{fD}}\\ p_{\mathrm{fwD}}\\ q_{\mathrm{fwD}}\end{bmatrix}=\begin{bmatrix}R_1\\ R_2\\ R_3\end{bmatrix} \quad (19)$$

4. MODEL VALIDATION

This model is validated by comparing the typical well test curve of penetrating inclined fractures published by Dinh and Tiab [15]. Since the fracture conductivity in their model is infinite, we set the F_{cD} in our model as 10^3 to character the infinite conductivity approximately.

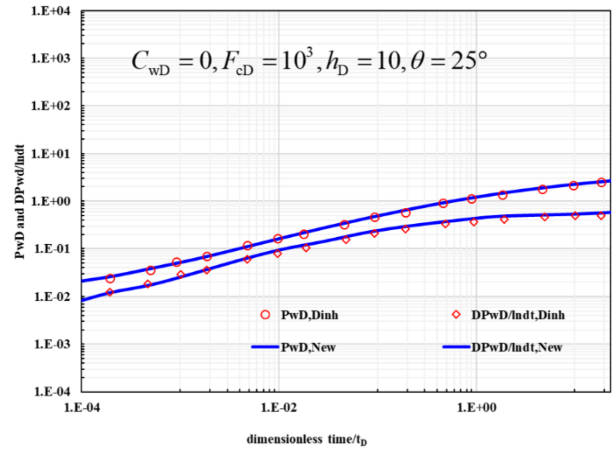

Fig. 2 Comparison of the result of semi-analytical model with the result Dinh

As shown in Fig. 2, the result of our model matched well with that of Dinh.

5. RESULTS AND DISCUSSION

5.1 *Transient-Pressure Behavior*

In practice, the flow regimes of PPIF networks are heavily influenced by the parameters of PPIF and reservoir matrix. Fig. 3 shows the relatively complete flow regimes of single finite conductivity inclined hydraulic fracture.

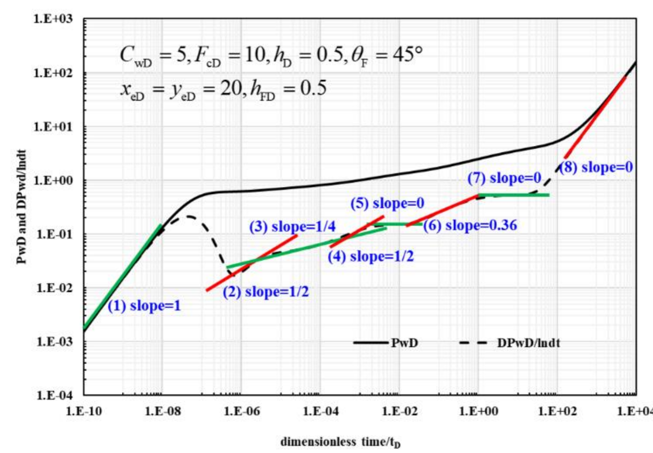

Fig. 3 Flow regimes of fractured well with partially penetrating inclined fracture

There are 8 flow regimes can be observed, include (1) wellbore afterflow, (2) Fracture linear flow, (3) Bilinear flow, (4) Formation linear flow, (5) Early radial flow, (6) Compound linear flow, (7) Pseudo-radial flow, (8) Boundary-dominated flow.

The changes of flow regimes can be observed when the inclined hydraulic fracture intersects with natural fractures. As shown in Fig. 4, with the influence of nature fracture, the flow in fracture system last longer. The bilinear flow regime is prolonged, and the early radial flow regime is absent at this condition. This result means with the increasing complexity of fracture system, the during time of fracture flow last longer and need more time to achieve steady-state flow in fracture system. During this time, the regimes in the matrix would change from linear flow to radius flow.

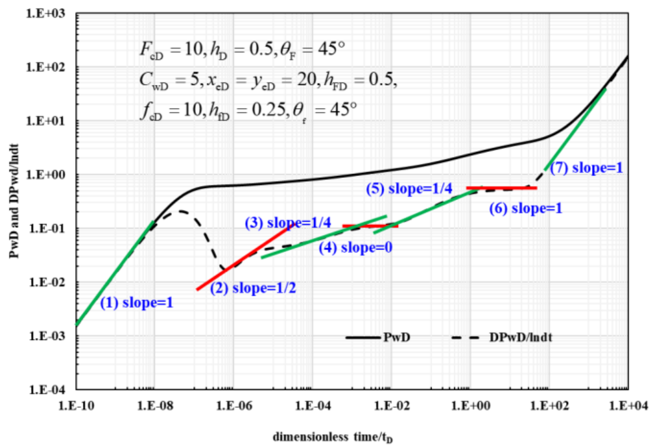

Fig. 4 Flow regimes of fractured well with partially penetrating inclined fracture and nature fractures

5.2 PARAMETER SENSITIVITY ANALYSIS

The fracture height and incline angle are two main factors of fracture spatial distribution. The influence on the typical curves were analyzed.

Fig. 5 shows the influence of nature fracture height on transient behaviors. The fracture height mainly influences the middle periods. With the increase of the fracture penetration ratio, the formation linear flow and early radial flow regime will be delayed. Especially, when the penetration ratio is large (> 0.6), the early radial flow regime will cannot be observed. This indicates when the nature fracture height is small, the flow in the matrix system will easier achieve an early radial flow, and with the increase of nature fracture height, the time will delay.

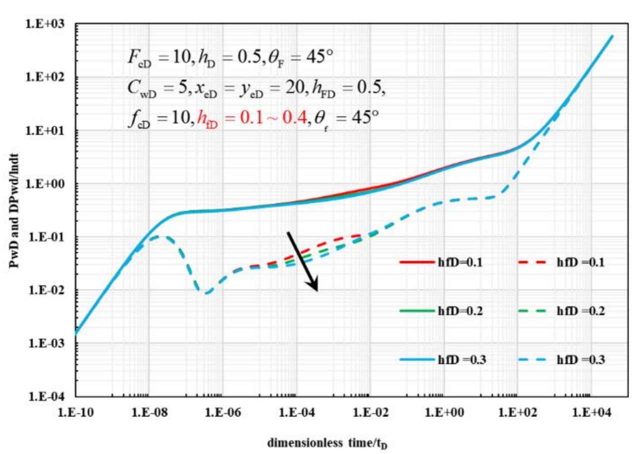

Fig. 5 Effect of dimensionless nature fractures height on transient responses of PPIF networks

The incline angle of fractures is the angle between the surface of fracture and the vertical direction. As shown in Fig. 6, the incline angle mainly influences the early and medium flow regimes. With the increase the inclined angle, the bilinear flow regime appears earlier, and this period will last longer. When the incline angle is large enough (>60°), the bilinear period would cover the early radio period.

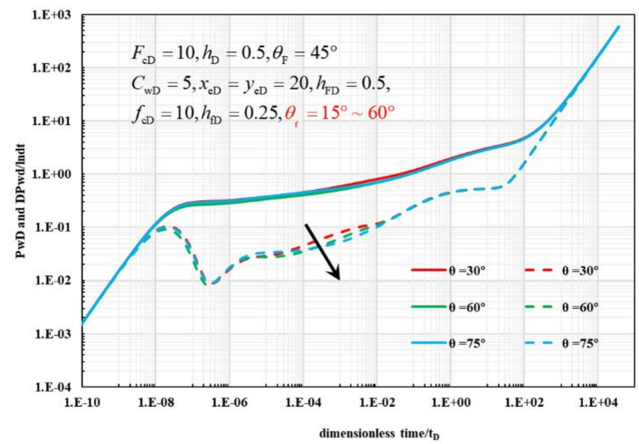

Fig. 6 Effect of incline angle of nature fractures on transient responses of PPIF networks

6. CONCLUSIONS

(1) A novel semi-analytical method for MHFW with PPIF networks is proposed. The use of triangular elements makes it easier to character the geometry of fractures, and the introduce of numerical integral technology for spatial triangle decrease the calculation complexity of this model greatly.

(2) There are eight main flow regimes for the hydraulic partially penetrating inclined fractures. When the hydraulic fractures intersect with the nature fractures, the duration of bilinear flow, formation linear flow and early radial flow will change, and the condition of natural fracture can be analyzed according to the flow regimes preliminary.

(3) With the increase of the nature fracture height and incline angle, the bilinear flow period will last longer. When the penetration ratio (> 0.6) or the incline angle (>60°) is large enough, the early radial flow will be absent.

ACKNOWLEDGEMENT

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